# Raman and Thompson regimes of amplification in a wiggler with noncollinear laser and electron beams

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The collective and single-electron amplification regimes of a noncollinear free-electron laser (FEL) are studied within the framework of dispersion equations. In the limit of small-signal gain the growth rates and the conditions for self-amplified excitations are found for the collective (Raman) and single-electron (Thompson) regimes. The Raman regime is shown to be preferable for the coherent spontaneous second harmonic generation by ultrarelativistic electron beams. Raman excitations in a noncollinear FEL, e.g., in an FEL without inversion, are favored by the noncollinear geometry of the electron and the laser beams, and by the relativity of the beam electrons.

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## I. INTRODUCTION

In a free-electron laser (FEL) [1,2], the accelerated motion of electrons in the ponderomotive potential, formed by the combined field of the wiggler and the electromagnetic wave, produces coherent stimulated radiation. Under the influence of the ponderomotive potential, a grating in the spatial density of electrons (bunching) on the scale of the electromagnetic wavelength is produced. As a result, an amplification of the electromagnetic wave (laser action) occurs.

It is common for a FEL's setup to have the electron beam aligned along the amplified electromagnetic beam and the wiggler axis. Recent years have seen increased interest in applying the noncollinear arrangement of electron and laser beams to various devices.

For example, the so-called free-electron laser without inversion (FELWI) proposed recently [3–8] relies on the noncollinear arrangement of the electron and laser beams. A FELWI aims to improve performance of FELs and optical klystrons by advanced usage of laser-induced electron phasing in the first wiggler, which is specific to the noncollinear geometry and reveals itself in correlation of the laser-induced changes of the electron energy and the transverse velocity. The new geometry promises to extend FEL operation to shorter wavelengths. It was shown [3–8] that one can control the time of electron entrance to the second wiggler so that the gain $G$ of an FELWI as a function of the detuning $\Omega = \omega(v_0 - v_{res})/c$ from the resonance condition is mostly positive, so that $\int G(\Omega) d\Omega > 0$.

New noncollinear schemes have been suggested for high-power short-wavelength coherent spontaneous radiation production as well as for seeding short-wavelength FELs [9]. A FEL noncollinearity was shown to assist the transverse velocity modulation of an electron beam and the efficiency of electron bunching, which resulted in some orders of magnitude enhancement of the coherent spontaneous radiation power and its harmonics. Noncollinearity in the modulator enabled separation of the radiation and the electron beam without bending magnets thus avoiding the problem of nonconservation of transverse bunching at bending magnets.

Note that the interest to apply the noncollinear arrangement of electron and laser beams in a FEL has not arisen in recent years. For example, in [10] the theory of free-electron lasers is extended in a single electron approximation to include the coupling between an electron beam and optical wave propagating at an angle in an arbitrary harmonic. Later this result was generalized for even harmonic generation and for the case of transverse field variation [11].

Concerning the laser acceleration problem in the ponderomotive potential, formed by the combined field of the wiggler and the electromagnetic wave, the classical interaction of charged particles propagating in an undulator at an angle to a strong electromagnetic wave was examined in [12]. The effect of "reflection" and capture in such a field has been found, which is conditioned by the direct and inverse stimulated undulator effect.

Besides, the noncollinear arrangement of an inverse FEL (IFEL) [13] was used to accelerate the electrons in a wide energy range. The noncollinear geometry provided compensation for phase slippage at the focal volume. Note that the acceleration process in an IFEL is described similarly to the well-known FEL equations.

To analyze simultaneously Thompson and Raman amplification regimes for a noncollinear arrangement of the laser and electron beams is interesting from various points of view. Such an analysis can also yield a unified view of different results obtained so far. A FELWI operation [3–8] has been analyzed so far using the single-electron approximation (Thompson regime): the propagation of the only electron through the FEL system was considered and the resulting gain was averaged over the electron distribution. However, it is known that the change of the system geometry may influence the kind of excitation regime [14], i.e., it can lead to a change from the single-electron amplification regime to the collective one and vice versa. The collective amplification regime in a noncollinear wiggler loaded with an *overdense* homogeneous plasma was analyzed [15] when the electromagnetic (em) wave frequency was below the plasma frequency. It has been suggested [15] that considered effect will apply in Raman FELs to produce radiation in ir and far-ir regions.

The aim of the current paper is to study in detail the amplification in a single wiggler for a noncollinear arrangement of the electron beam and the amplified wave in order to generalize the previous results and to clarify the limitations of the single-electron approach used for FEL with a noncollinear arrangement [9], in particular, for FELWIs (papers [3–8]). The analysis presented in the paper for a FEL at high electron densities generalizes the approaches and results in [14] (and in references cited therein) and the results obtained for FELs with high-density of electron beam [15].

The paper is organized as follows. In Sec. II, we formulate the main assumptions and derive the self-consistent system of the nonlinear evolutionary equations which determine the stimulated radiation in the wiggler and the electrons' dynamics. We apply the approach developed in plasma electronics [14] that naturally describes the beam oscillations. The model describes both the *linear* and the *nonlinear regimes* of the FEL instability. In Sec. III, the small-signal gain analysis is examined to obtain the growth rates and its conditions of excitations. In Sec. IV, we investigate the nonlinear regimes of instability and its saturation. In the Conclusion, we summarize the results and compare them to the previous ones.

## II. BASIC EQUATIONS

We consider the induced radiation by the monoenergetic beam of electrons entering the wiggler with the same velocity. We choose the coordinate system so that the axis $0z$ coincides with the axis of the wiggler while the wiggler vector-potential is parallel to the axis $0y$. We assume that the static magnetic field of a plane wiggler $\mathbf{A}_w$ is independent on the transverse coordinates $x$ and $y$. Also we approximate the static magnetic field by a harmonic function

$$\mathbf{A}_w = A_w \mathbf{e}_y, \quad \text{where } A_w = A_0 e^{-i\mathbf{k}_w \cdot \mathbf{r}} + \text{c.c.}, \tag{1}$$

where $\mathbf{k}_w = (0,0,k_w)$ is the wiggler wave vector; "c.c." denotes the complex conjugation, and $\mathbf{e}_y$ is the unit vector along the $y$ axis. The wiggler field causes an electron to oscillate along the $y$ axis. For this reason, the electron interacts most efficiently with a light wave if the latter is linearly polarized. Next we assume that the vector potential of the laser wave has a linear polarization $\mathbf{A}_L = A_L(t,x,z)\mathbf{e}_y$. In this case, the vector potential $\mathbf{A}_L$ defines the pure vortex part of field $\vec{\nabla} \cdot \mathbf{A}_L = 0$, while the scalar potential $\phi = \phi(t,x,z)$ defines longitudinal beam waves in the system. The Maxwell equations can be written

$$\Delta_\parallel \phi \equiv (\partial_x^2 + \partial_z^2)\phi = -4\pi\rho, \tag{2}$$

$$(c^2\partial_x^2 + c^2\partial_z^2 - \partial_t^2)A_L = -4\pi c j_y. \tag{2'}$$

The electron beam entering the wiggler is assumed to have a uniform density $n_b$ and no spread in the electron velocity $\mathbf{u} = (-u\sin\alpha; 0; u\cos\alpha)$. Then the initial distribution function can be written in the form $f_0 = n_b \delta(\mathbf{p}_0 - m\gamma_0 \mathbf{u})$. Here $e$ and $m$ are the charge and mass of an electron, and $\gamma$ is the Lorentz factor. To integrate over the initial coordinates with this initial distribution function gives the charge and current densities for beam with charge compensated (see Appendix A)

$$\rho = en_b \left\{ \int \delta[x - x(t,x_0,z_0)]\delta[z - z(t,x_0,z_0)]dx_0 dz_0 - 1 \right\}, \tag{3}$$

$$j_y = en_b \int v_y(t,x_0,z_0)\delta[x - x(t,x_0,z_0)] \times \delta[z - z(t,x_0,z_0)]dx_0 dz_0. \tag{3'}$$

Here $x(t,x_0,z_0)$ and $z(t,x_0,z_0)$ are solutions of Hamilton equations

$$\dot{\mathbf{r}} = \frac{\partial H}{\partial \mathbf{P}}, \quad \dot{\mathbf{P}} = -\frac{\partial H}{\partial \mathbf{r}} \tag{4}$$

with initial conditions $\mathbf{r}_\parallel(0) = \mathbf{r}_{\parallel 0}$, $\mathbf{p}_\parallel(0) = m\gamma_0 \mathbf{u}$. $\mathbf{P} = \mathbf{p} + \frac{e}{c}\mathbf{A}$ is the canonical momentum; $A = A_w + A_L$ is a sum of vector potentials. The Hamiltonian of the electron in the field

$$H = \sqrt{m^2c^4 + c^2\left(\mathbf{P} - \frac{e}{c}\mathbf{A}\right)^2} + e\phi = mc^2\gamma + e\phi \tag{5}$$

does not depend on $y$: $\partial H/\partial y = 0$, and so we obtain the first integral

$$v_y = -\frac{e}{mc}\frac{A(t,x,z)}{\gamma}\bigg|_{\substack{x=x(t,x_0,z_0)\\ z=z(t,x_0,z_0)}}. \tag{6}$$

We represent all vectors as sums of two components: the first component being in the plane $xz$ [designated as $\mathbf{f}_\parallel = (f_x, 0, f_z)$] and the second component being parallel to the vector-potential or vector $\mathbf{e}_y$ (designated as $f_y \mathbf{e}_y$). The Hamilton equations (4) determine the electron coordinate and velocity

$$\dot{\mathbf{r}}_\parallel = \frac{\partial H}{\partial \mathbf{P}_\parallel} = \mathbf{v}_\parallel, \tag{7}$$

$$\dot{\mathbf{v}}_\parallel = -\frac{e}{m\gamma}\left[\boldsymbol{\nabla}_\parallel - \frac{1}{c^2}\mathbf{v}_\parallel(\mathbf{v}_\parallel \cdot \boldsymbol{\nabla}_\parallel)\right]\phi - \frac{1}{2}\left(\frac{e}{mc}\right)^2 \times \frac{1}{\gamma^2}\left[\boldsymbol{\nabla}_\parallel + \frac{\mathbf{v}_\parallel}{c^2}\frac{\partial}{\partial t}\right]A^2. \tag{7'}$$

We introduce two relativistic factors

$$\gamma_\parallel = \left(1 - \frac{v_\parallel^2}{c^2}\right)^{-1/2}, \quad \gamma = \gamma_\parallel\left[1 + \frac{1}{c^2}\left(\frac{e}{mc}\right)^2 A^2\right]^{1/2}. \tag{8}$$

The field equations (2) take the form

$$\Delta_\parallel \phi = -\frac{m}{e}\omega_b^2 \left\{ \int \delta[\mathbf{r}_\parallel - \mathbf{r}_\parallel(t,\mathbf{r}_{\parallel 0})] d\mathbf{r}_{\parallel 0} - 1 \right\}, \tag{9}$$

$$(c^2\Delta_\parallel - \partial_t^2)A_L - \omega_b^2 \int \frac{A_L}{\gamma} \delta[\mathbf{r}_\parallel - \mathbf{r}_\parallel(t,\mathbf{r}_{\parallel 0})] d\mathbf{r}_{\parallel 0} = \omega_b^2 \int \frac{A_{\mathrm{w}}}{\gamma} \delta[\mathbf{r}_\parallel - \mathbf{r}_\parallel(t,\mathbf{r}_{\parallel 0})] d\mathbf{r}_{\parallel 0}. \tag{9'}$$

Here $\omega_b^2 = 4\pi e^2 n_b/m$ is a square of Langmuir frequency of the electron beam; here and below $\gamma = \gamma(t,\mathbf{r}_{\parallel 0})$.

We seek the solutions for field in the forms

$$\phi = \frac{1}{2}[\psi e^{i\mathbf{k}_0\cdot\mathbf{r}_\parallel} + \mathrm{c.c.}], \tag{10}$$

$$A_L = A_+ e^{i(\mathbf{k}_0 - \mathbf{k}_{\mathrm{w}})\mathbf{r}_\parallel} + A_- e^{-i(\mathbf{k}_0 + \mathbf{k}_{\mathrm{w}})\mathbf{r}_\parallel}. \tag{10'}$$

Here the vector $\mathbf{k}_0 = k_0(\sin\theta, 0, \cos\theta)$ lies on plane $xz$. To facilitate the solutions, we introduce the dimensionless coordinates as $\xi = \mathbf{k}_0\mathbf{r}_\parallel$, $\xi_0 = \mathbf{k}_0\mathbf{r}_{\parallel 0}$ and introduce the dimensionless spatial Fourier components of the electron charge and current density, $\sigma$ and $\hat{\sigma}$, respectively:

$$\sigma = \frac{1}{\pi}\int_0^{2\pi} e^{-i\xi} d\xi_0, \quad \hat{\sigma} = \frac{1}{\pi}\int_0^{2\pi} \frac{e^{-i\xi}}{\gamma} d\xi_0. \tag{11}$$

Note that the integration is performed over the laser wavelength. Here and below $\xi = \xi(t,\xi_0)$. Substituting the solutions (10) in Eqs. (9) and averaging these equations over the electrons on wavelength (integrating over wavelength), we get

$$\phi = \frac{1}{2}\frac{m}{e}\frac{\omega_b^2}{k_0^2}[\sigma e^{i\xi} + \mathrm{c.c.}], \tag{12}$$

$$\frac{d^2A_+}{dt^2} + \omega_+^2 A_+ + \omega_b^2 I_0 A_- = -\frac{1}{2}\omega_b^2 \hat{\sigma} A_0, \tag{12'}$$

$$\frac{d^2A_-}{dt^2} + \omega_-^2 A_- + \omega_b^2 I_0^* A_+ = -\frac{1}{2}\omega_b^2 \hat{\sigma}^* A_0, \tag{12''}$$

where

$$\omega_\pm^2 = (\mathbf{k}_0 \mp \mathbf{k}_{\mathrm{w}})^2 c^2 + \omega_b^2 \langle \gamma^{-1} \rangle,$$

$$I_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-2i\xi(t,\xi_0)}}{\gamma(t,\xi_0)} d\xi_0,$$

$$\langle \gamma^{-1} \rangle = \frac{1}{2\pi}\int_0^{2\pi} \frac{d\xi_0}{\gamma(t,\xi_0)}. \tag{13}$$

Equations (12′) and (12″) are equations of stimulated oscillations of two coupled systems: the electron beam and the amplified electromagnetic field.

The equations of electron motion take the form

$$\dot{\mathbf{r}}_\parallel = \mathbf{v}_\parallel, \tag{14}$$

$$\dot{\mathbf{v}}_\parallel = -\frac{i}{2}\frac{\omega_b^2}{k_0^2}\frac{1}{\gamma}\left[\mathbf{k}_0 - \frac{1}{c^2}\mathbf{v}_\parallel(\mathbf{k}_0\mathbf{v}_\parallel)\right]\sigma e^{i\xi} - \left(\frac{e}{mc}\right)^2 \frac{e^{i\xi}}{\gamma^2}\left(i\mathbf{k}_0 + \frac{\mathbf{v}_\parallel}{c^2}\frac{d}{dt}\right) \times (A_0^* A_+ + A_0 A_-^*) + \mathrm{c.c.}, \tag{14'}$$

with the initial conditions $\mathbf{r}_\parallel(t=0) = \mathbf{r}_{\parallel 0}$, $\mathbf{v}_\parallel(t=0) = \mathbf{u}$. The self-consistent system of Eqs. (11)–(14) defines stimulated radiation in the wiggler and describes both *linear* and *nonlinear regimes* of the FEL instability.

## III. SMALL SIGNAL GAIN

### A. Dispersion equation

Let us consider the *linear stage of instability* (small signal gain). We linearize Eqs. (11) and (14′) for small perturbations $\delta\mathbf{r}$, $\delta\mathbf{v}$, which are proportional to the amplitudes of the laser waves $A_\pm$. All values are expanded in sums of nondisturbed and disturbed components: $\mathbf{r}_\parallel = \mathbf{r}_{\parallel 0} + \mathbf{u}t + \delta\mathbf{r}_\parallel$ or $\xi = \xi_0 + \mathbf{k}_0\cdot\mathbf{u}t + \mathbf{k}_0\delta\mathbf{r}_\parallel$, $\mathbf{v}_\parallel = \mathbf{u} + \delta\mathbf{v}_\parallel$, $\omega = \mathbf{k}_0\cdot\mathbf{u} + \Delta_\omega$, $\gamma = \gamma_0 + \delta\gamma$, and $\gamma_\parallel = \gamma_{\parallel 0} + \delta\gamma_\parallel$. Here

$$\gamma_0 = \gamma_{\parallel 0}\sqrt{1+\mu}, \quad \gamma_{\parallel 0} = (1-\beta^2)^{-1/2}, \tag{15}$$

where $\beta = u/c$. The wiggler parameter $\mu$, which will play a significant role, is defined as a dimensionless square of the wiggler field amplitude

$$\mu = \frac{2}{c^2}\left(\frac{e}{mc}\right)^2 |A_0|^2. \tag{16}$$

By linearizing equations over small perturbations, we obtain $I_0 = 0$ and

$$\sigma = \delta\sigma e^{-i\mathbf{k}_0\cdot\mathbf{u}t}, \quad \delta\sigma = \frac{1}{\pi}\int_0^{2\pi} (-i\mathbf{k}_0\cdot\delta\mathbf{r}_\parallel) e^{-i\xi_0} d\xi_0, \tag{17}$$

$$\hat{\sigma} = \delta\hat{\sigma} e^{-i\mathbf{k}_0\cdot\mathbf{u}t}, \quad \delta\hat{\sigma} = \frac{\delta\sigma}{\gamma_0} - \frac{1}{\pi\gamma_0}\int_0^{2\pi} \frac{\delta\gamma}{\gamma_0} e^{-i\xi_0} d\xi_0. \tag{17'}$$

For the small signal gain the vector potential is a harmonic function of time

$$A_\pm = a_\pm e^{\mp i\omega t}. \tag{18}$$

The frequency $\omega$ is complex and its imaginary part defines the growth rate of the FEL instability.

The solution to the linearized equations of motion (14′) follows:

$$\delta\mathbf{v}_\parallel = \left(\frac{e}{mc}\right)^2 \frac{e^{i\xi_0}}{D_b\gamma_0^3}\left(\beta_1\mathbf{k}_0 - \frac{\omega}{c^2}\beta_2\mathbf{u}\right) \times (A_0^* a_+ + A_0 a_-^*) e^{-i\Delta_\omega t} + \mathrm{c.c.}, \tag{19}$$

$$\delta\mathbf{r}_\parallel = i\left(\frac{e}{mc}\right)^2 \frac{e^{i\xi_0}}{D_b\gamma_0^3\Delta_\omega}\left(\beta_1\mathbf{k}_0 - \frac{\omega}{c^2}\beta_2\mathbf{u}\right) \times (A_0^* a_+ + A_0 a_-^*) e^{-i\Delta_\omega t} + \mathrm{c.c.} \tag{19'}$$

Here

$$D_b = (\omega - \mathbf{k}_0 \cdot \mathbf{u})^2 - \Omega_b^2 \tag{20}$$

is the dispersion function of electron beam wave associated with the beam frequency $\Omega_b$, where

$$\Omega_b^2 = \frac{\omega_b^2}{\gamma_0}\left[1 - \frac{(\mathbf{k}_0 \cdot \mathbf{u})^2}{k_0^2 c^2}\right]. \tag{21}$$

The coefficients $\beta_1$ and $\beta_2$ equal

$$\beta_1 = \gamma_0[\omega - (\mathbf{k}_0 \cdot \mathbf{u})] - \frac{\omega_b^2(\mathbf{k}_0 \cdot \mathbf{u})}{k_0^2 c^2},$$

$$\beta_2 = \gamma_0[\omega - (\mathbf{k}_0 \cdot \mathbf{u})] - \frac{\omega_b^2}{\omega}. \tag{22}$$

The perturbations of the dimensionless charge density $\sigma$ and the dimensionless current density $\hat{\sigma}$ follow:

$$\delta\sigma = 2\left(\frac{e}{mc}\right)^2 \frac{1}{D_b \gamma_0^2}\left(k_0^2 - \frac{(\mathbf{k}_0 \cdot \mathbf{u})\omega}{c^2}\right) \times (A_0^* a_+ + A_0 a_-^*) e^{-i\Delta_\omega t}, \tag{23}$$

$$\delta\hat{\sigma} = \frac{2}{c^2}\left(\frac{e}{mc}\right)^2 \frac{c^2 k_0^2 - \omega^2 + \omega_b^2 \gamma_0^{-1}}{D_b \gamma_0 \gamma_{\parallel 0}^2 (1+\mu)} \times (A_0^* a_+ + A_0 a_-^*) e^{-i\Delta_\omega t}. \tag{23$'$}$$

Substituting Eq. (23′) in field equations (12′) and (12″) we obtain the dispersion equation, which defines relation $\omega = \omega(\mathbf{k})$.

Let us consider the resonant case $\omega \approx \omega_+ = \mathbf{k}_0 \cdot \mathbf{u} - \Omega_b$, which corresponds to the maximal growth rate of the FEL instability. In this case $A_- = a_- = 0$. As a result the dispersion equation takes the simple form

$$D_b(\omega^2 - \omega_+^2) = \frac{1}{2}\omega_b^2 \frac{\mu}{1+\mu} \frac{c^2 k_0^2 - \omega^2 + \omega_b^2 \gamma_0^{-1}}{\gamma_0 \gamma_{\parallel 0}^2}. \tag{24}$$

Here

$$\omega_+^2 = (\mathbf{k}_0 - \mathbf{k}_w)^2 c^2 + \frac{\omega_b^2}{\gamma_0}. \tag{25}$$

The solution of the dispersion equation (24) under the resonant condition gives the frequency $\omega$

$$\omega = \omega_+ + \delta\omega = (\mathbf{k}_0 \cdot \mathbf{u}) - \Omega_b + \delta\omega. \tag{26}$$

The presence of the beam leads to the complex shift of frequency $\delta\omega$ (where $|\delta\omega| \ll \omega_+$).

For the resonant conditions described above, the dispersion function of the beam and the detuning of the frequency from the resonance are equal to $D_b = \delta\omega^2 - 2\delta\omega\Omega_b$, $\Delta_\omega = \delta\omega - \Omega_b$, respectively.

We introduce the complex dimensionless shift of frequency $\delta = \delta\omega/\Omega_b$. Then the dispersional equation (24) can be written in terms of $\delta$ as

$$\delta^2(\delta - 2) + \frac{1}{2}\frac{\mu}{1+\mu}\frac{\omega_b^2}{\Omega_b^2 \gamma_0 \gamma_{\parallel 0}^2}\delta = |q|, \tag{27}$$

where

$$|q| = \frac{1}{4}\frac{\mu}{1+\mu}\frac{(1+\nu)^2}{\nu}\left(\frac{k_0 c}{\mathbf{k}_0 \cdot \mathbf{u}}\right)^2 \frac{\omega_b^2}{\Omega_b^2 \gamma_0 \gamma_{\parallel 0}^2} \tag{28}$$

and

$$\nu = \frac{\omega_b^2}{\omega_+ \Omega_b \gamma_0}. \tag{29}$$

For a nonrelativistic beam ($\beta \ll 1$) the parameter $\nu$ reduces to the ratio of the frequencies $\nu = \omega_b/\omega_+$, i.e., to dimensionless Langmuir frequency. It is shown below that the parameter $\nu$ defines the normal or anomalous behaviors of the growth rate, while the parameter $|q|$ defines the regime of instability (Raman or Thompson).

Note that for *collinear FEL geometry*, when $\alpha = \theta = 0$, and relativistic electron beams we get $|q| \approx 0.25\mu/(1+\mu)(1+\nu)^2/\nu$, that is the parameter $|q|$ depends on $\gamma_{\parallel 0}$ only through the intermediary value $\nu$. To the contrary, for *noncollinear FEL geometry*, when $\alpha + \theta \neq 0$, and relativistic electrons the parameter $|q|$ will strongly depend on $\gamma_{\parallel 0}$. For $\gamma_{\parallel 0}\sin(\alpha+\theta) \gg 1$ we obtain the asymptotic

$$|q| \approx \frac{\mu}{1+\mu}\frac{(1+\nu)^2}{\nu}\frac{1}{\gamma_{\parallel 0}^2 \sin^2(2\alpha + 2\theta)}. \tag{30}$$

In addition, for collinear FEL geometry with $\gamma_{\parallel 0}$ increasing the parameter $\nu$ grows as a function $\nu = (\omega_b/\omega_+)\sqrt{\gamma_{\parallel 0}/(1+\mu)} \propto \sqrt{\gamma_{\parallel 0}}$, while for noncollinear FEL geometry under condition $\gamma_{\parallel 0}\sin(\alpha+\theta) \gg 1$ the parameter $\nu$ drops as $\gamma_{\parallel 0}$ increases, namely $\nu = \omega_b/[\omega_+\sqrt{\gamma_0}\sin(\alpha+\theta)] \propto 1/\sqrt{\gamma_{\parallel 0}}$. This distinction leads to different dependance of the parameter $|q|$ on $\gamma_{\parallel 0}$: while for collinear laser geometry we have $|q| \sim \sqrt{\gamma_{\parallel 0}}$ (for $\nu \ll 1$) and $|q| \sim 1/\sqrt{\gamma_{\parallel 0}}$ (for $\nu \gg 1$), then for noncollinear laser geometry under ultrarelativistic conditions $\gamma_{\parallel 0}\sin(\alpha+\theta) \gg 1$ we have $|q| \sim \gamma_{\parallel 0}^{-3/2}$. As was shown later, this means that for relativistic electron beams $[\gamma_{\parallel 0}\sqrt{\nu}\sin(\alpha+\theta) \gg 1$ for $\nu \ll 1$ and $\gamma_{\parallel 0}\sin(\alpha+\theta)/\sqrt{\nu} \gg 1$ for $\nu \gg 1]$ propagating at a small angle to laser wave direction, *the collective amplification is possible for any value of parameter* $\mu$ (as distinct from collinear wiggler geometry [14], for which the Raman regime is absent for $\mu > 1$), that is for any lateral relativistic velocity of electrons. Now we consider different regimes of excitation.

### B. Collective amplification

For the *collective regime*, when $|\delta\omega| \ll \Omega_b$ or $|\delta| \ll 1$, and for the relativistic beam $\gamma_{\parallel 0}\sin(\alpha+\theta) \gg 1$ the dispersion equation (27) reduces to the quadratic form

$$\delta^2 - \frac{1}{4}\frac{\mu}{1+\mu}\frac{\delta}{\gamma_{\parallel 0}^2 \sin^2(\alpha+\theta)} + \frac{1}{2}|q| = 0 \tag{31}$$

leading to the growth rate for the collective regime: $\mathrm{Im}(\delta) = \sqrt{|q|/2}$ (see Appendix B) or

$$\mathrm{Im}(\delta\omega) = \frac{1}{2}\sqrt{\frac{\mu}{2(1+\mu)}}\frac{k_0 c}{\mathbf{k}_0 \cdot \mathbf{u}}\frac{\sqrt{\Omega_b \omega_+}}{\gamma_{\parallel 0}}\left(1 + \frac{\omega_b^2}{\omega_+ \Omega_b \gamma_0}\right). \tag{32}$$

The condition for Raman (collective) amplification can be rewritten as $|q| \ll 1$. Thus for *noncollinear FEL geometry* under relativistic condition $\gamma_{\|0} \sin(\alpha+\theta) \gg 1$ the collective regime holds for any lateral relativistic velocity of electrons. The increasing of the longitudinal velocity (or relativistic factor $\gamma_{\|0}$) for the *noncollinear FEL geometry* decreases the parameter $|q|$ and thus leads to the collective regime of amplification, independently from the value of the wiggler parameter $\mu$.

Consider asymptotic formulas for growth rates of the undulator radiation in the case of ultrarelativistic electron beams, $\gamma_{\|0} \sin(\alpha+\theta) \gg 1$:

$$\mathrm{Im}(\delta\omega) = \begin{cases} \frac{1}{2}\sqrt{\frac{\mu}{2}}\frac{\sqrt{\omega_+\omega_b \sin(\alpha+\theta)}}{\gamma_0^{5/4}\cos(\alpha+\theta)}, & \frac{\mu}{1+\mu}\frac{1}{\gamma_{\|0}^2 \sin^2(\alpha+\theta)} \ll \nu \ll 1 \\ \frac{1}{2}\sqrt{\frac{\mu}{2}}\frac{\gamma_0^{-7/4}\omega_b^{3/2}}{\cos(\alpha+\theta)\sqrt{\omega_+ \sin(\alpha+\theta)}}, & 1 \ll \nu \ll \frac{1+\mu}{\mu}\gamma_{\|0}^2 \sin^2(\alpha+\theta). \end{cases} \tag{33}$$

The first growth rate (33) is the usual one [14] for collective regimes, since its dependence on Langmuir beam frequency is $\omega_b^{1/2}$. The second growth rate is described by dependence $\omega_b^{3/2}$. This anomalous behavior is a result of energy phase equalizing, which takes place both in collinear [14] and noncollinear wiggler geometry. For a noncollinear FEL geometry the growth rate depends on the geometric parameter $\sin(\alpha+\theta)$. Note that the condition $\nu \gg 1$ can hold for an overdense ultrarelativistic beam, when $(\omega_b/\omega_+)^2 \gg \sin(\alpha+\theta)/\sqrt{1+\mu}$. The condition of the amplification with the second growth rate of Eq. (33) can be written in the form

$$\max\left\{1,\left(\frac{\mu}{(1+\mu)^{3/4}}\frac{\omega_b}{\omega_+}\frac{1}{\sqrt{\sin(\alpha+\theta)}}\right)^{2/5}\right\} \ll \gamma_{\|0}\sin(\alpha+\theta) \ll \frac{\omega_b}{\omega_+}\frac{(1+\mu)^{1/4}}{\sqrt{\sin(\alpha+\theta)}}. \tag{34}$$

The formula (33) shows that the increasing of longitudinal velocity (or relativistic factor $\gamma_{\|0}$) for noncollinear wiggler geometry leads to excitation of the collective regime independently from the values $\mu$ and $\nu$.

### C. Single-electron amplification

For the *single-electron amplification* (Thompson regime) the frequency shift $|\delta\omega|$ is larger than beam frequency, namely $|\delta\omega| \gg \Omega_b$ or $|\delta| \gg 1$, and the dispersion equation (27) is cubic

$$\delta^3 + \frac{1}{2}\frac{\mu}{1+\mu}\frac{\omega_b^2}{\Omega_b^2\gamma_0\gamma_{\|0}^2}\delta - |q| = 0. \tag{35}$$

The solution Eq. (35), being written for the image part of $\delta$, is (see Appendix C)

$$\mathrm{Im}(\delta) = \frac{\sqrt{3}}{2}|q|^{1/3}. \tag{36}$$

The above definition of Thompson type of amplification ($|\delta| \gg 1$) can be rewritten as $|q| \gg 1$.

Consider the asymptotic of the growth rate $\mathrm{Im}(\delta\omega)$ for $\gamma_{\|0}\sin(\alpha+\theta) \gg 1$, the case of interest for optical klystron, and particular for FELWI, applications. Under conditions

$$\nu \ll \min\left\{\frac{\mu}{1+\mu}\frac{1}{\gamma_{\|0}^2\sin^2(\alpha+\theta)},1\right\} \tag{37}$$

the asymptotic behavior of the growth rate is

$$\mathrm{Im}(\delta\omega) = \frac{\sqrt{3}}{2^{5/3}}\left[\frac{\mu}{(1+\mu)^2}\frac{\omega_b^2\omega_+}{\gamma_{\|0}^3}\tan^2(\alpha+\theta)\right]^{1/3}. \tag{38}$$

For very large $\nu$, namely $\nu \gg \gamma_{\|0}^2\sin^2(\alpha+\theta)(1+\mu)/\mu$, the growth rate of single-electron amplification has the anomalous behavior

$$\mathrm{Im}(\delta\omega) = \frac{\sqrt{3}}{2^{5/3}}\frac{\mu^{1/3}}{\sqrt{1+\mu}}\left(\frac{\omega_b}{\omega_+}\right)^{1/3}\frac{\omega_b}{\gamma_{\|0}^{4/3}\cos^{2/3}(\alpha+\theta)}. \tag{39}$$

As for *collinear FEL geometry*, here the growth rate depends on Langmuir frequency of the electron beam as $\omega_b^{4/3}$ and is almost independent from the angle between the electron beam and the laser wave.

However, the realization of this amplification regime using an ultrarelativistic beam is almost impossible because of the large required charge of beam $(\omega_b/\omega_+) \gg \gamma_{\|0}^{5/2}\sin^3(\alpha+\theta)(1+\mu)^{3/4}/\mu$, and as a consequence it is a necessary very big current, which is limited by *vacuum current* for vacuum devices.

The above calculations indicate that if the wiggler is loaded with the noncollinear electron beam and the laser wave, then the Raman-type amplification is feasible for relatively small densities of the electron beam [the first growth rate in Eq. (33)]. We find that the electron current density required for Raman-type amplification drops with increasing the relativistic factor $\gamma_{\|0}$ of the beam. This means that *collective amplification* can be realized in optical wigglers, in particular, in FELWI, in which the ultrarelativistic noncollinear beams are used.

In Figs. 1 and 2 the dependencies of growth rate and parameter $\delta$ are presented as the functions of beam current

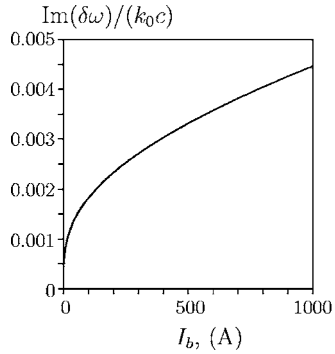


FIG. 1. The growth rate as a function of beam current.

for the following parameters, being used in paper [8]: $\alpha=0.13$, $\theta=0$, $\gamma=15$, rms beam radius $r=70\ \mu$m, laser wavelength $\lambda_L=359\ \mu$m, period of the wiggler magnets $\lambda_w=2.72$ cm, and normalized wiggler field $\mu=0.80645$.

### D. Transverse bunching

Finally, consider the effect of the transverse bunching of the beam, which can lead to *beam filamentation*. Using Eq. (2), and Eqs. (12), (17), and (23), the perturbation of beam density is found

$$\frac{\delta n}{n_b} = \left(\frac{e}{mc}\right)^2 \frac{1}{D_b \gamma_0^2}\left(k_0^2 - \frac{(\mathbf{k}_0 \cdot \mathbf{u})\omega}{c^2}\right) A_0^* a_+ e^{i(\xi-\omega t)} + \text{c.c.} \tag{40}$$

Since $\mathbf{k}_0 \| \mathbf{u}$ the transverse modulation of the beam density is formed in the direction, which is perpendicular to the beam velocity in the $xy$ plane. The spatial period of this transverse modulation equals

$$\lambda_b = \frac{2\pi}{k_0}\sin(\alpha+\theta). \tag{41}$$

For the normal Raman regime [see the first Eq. (33)] the density modulation $\delta n$ has the order $n_b^{1/4}$, while for normal Thompson regime [see Eq. (38)] $\delta n \sim n_b^{1/3}$. The velocity variation in the transverse direction to the beam velocity is

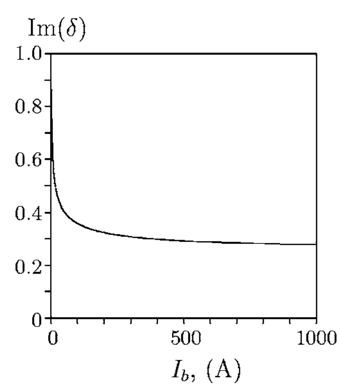


FIG. 2. The dependance of $\delta$ from beam current.

$$\delta \mathbf{v}_{\text{perp}} = \left(\frac{e}{mc}\right)^2 \frac{\beta_1 \mathbf{k}_{0\perp}}{D_b \gamma_0^3}(A_0^* a_+ + A_0 a_-^*) e^{i(\xi-\omega t)} + \text{c.c.}, \tag{42}$$

where $\mathbf{k}_{0\perp} = \mathbf{k}_0 - (\mathbf{k}_0 \cdot \mathbf{u})\mathbf{u}/u^2$. The current density $\delta \mathbf{j} = e\,\delta n\,\delta \mathbf{v}_{\text{perp}}$ has the second harmonic term. This effect as was shown in a single electron approximation [9], can be used for generation of the second harmonic of the laser radiation. Since the transverse modulation depends on the initial phase $\xi_0$, the emission on second harmonic can be only coherent spontaneous radiation (CSR).

We estimate the current density $\delta \mathbf{j}$ assuming as above that $\gamma_{\|0}\sin(\alpha+\theta) \gg 1$. For Raman type excitation we have $\delta \mathbf{j} \sim \mu(a_+/A_0)^2 k_0 c \gamma_0^{-1} \mathbf{k}_{0\perp}$ for a weak beam current, when $\omega_b/(k_0 c) \ll \sqrt{\gamma_0}\sin(\alpha+\theta)$. This means that the amplitude of the second harmonic radiation does not depend on the beam density $n_b$ and is proportional to $\gamma_0^{-1}$. For a dense beam, when $\omega_b/(k_0 c) \gg \sqrt{\gamma_0}\sin(\alpha+\theta)$, the transverse perturbation of the beam current $\delta \mathbf{j}$ depends on the current density as $\delta \mathbf{j} \sim \mu(a_+/A_0)^2 \omega_b \gamma_0^{-3/2} \mathbf{k}_{0\perp}/\sin(\alpha+\theta)$, i.e., is proportional to $\gamma_0^{-3/2}$ and does not depend on $(\alpha+\theta)$. For the Thompson regime of excitation we obtain the current perturbation on the second harmonic $\delta \mathbf{j} \sim \mu\sqrt{1+\mu(a_+/A_0)^2} k_0 c \gamma_0^{-1} \mathbf{k}_{0\perp}$, which does not depend on the beam current too.

The independence of $\delta \mathbf{j}$ from the beam density $n_b$ (or the beam current density) for the Thompson regime has a simple explanation. With the beam density $n_b$ dropping, the condition for one-particle Cherenkov resonance $\omega = (\mathbf{k}_0 \cdot \mathbf{u})$ holds true more accurately and the amplitude of the velocity modulation $\delta \mathbf{v}_{\text{perp}}$ grows as $n_b^{-1/3}$, making $\delta \mathbf{j}$ independent on the current density $\mathbf{j}$. Under the Cherenkov condition $\delta \mathbf{v}_{\text{perp}}$ becomes formally equal to infinity; this corresponds to infinite to time of the particle acceleration by the laser wave. The approach considered above is valid until $\delta v_{\text{perp}}/u \ll 1$ holds true. This condition puts the restriction from below on the beam density, which written for beam Langmuir frequency under the Thompson regime has the form $\omega_b \gg \mu(a_+/A_0)^{3/2}(k_0 c/\gamma_0)^{3/2}\sqrt{\sin(\alpha+\theta)/\omega_+}$. In the opposite case it is necessary to take into account the nonlinear terms leading to the saturation of perturbations. But there is an even stronger restriction. We assumed above that the electron interacts with the laser wave infinitely long. However, if $L$ is the length of the wiggler, the time of electron-wave interaction is limited by $t_{\max} = L/u$. The condition $\Delta_\omega t_{\max} \gg 1$ permits us to consider transverse motion of electrons as oscillations. For Thompson excitation this condition puts the limitation from below on the beam frequency:

$$\omega_b \gg \frac{(c\gamma_0/L)^{3/2}}{\sqrt{\mu\omega_+}\sin(\alpha+\theta)}. \tag{43}$$

If the condition (43) does not hold (i.e., $\Delta_\omega t_{\max} \ll 1$) we obtain another nonharmonic current density modulation $\delta \mathbf{j}$ for the Thompson regime

$$\delta\mathbf{j} = \frac{2^{7/3}}{3} e n_b c^4 \mu^{4/3} \left(\frac{a_+}{A_0}\right)^2 (1+\mu)^{1/3} \times \frac{k_0^2 \mathbf{k}_{0\perp}}{\gamma_0^2 \omega_b^{4/3} \omega_+^{2/3}} \sin^{2/3}(\alpha+\theta) t, \quad 0 < t < t_{\max}. \quad (44)$$

In this case we have the following estimation: $\delta\mathbf{j} \sim n_b^{1/3} \mu^{4/3} \gamma_0^{-2} \sin^{2/3}(\alpha+\theta) \mathbf{k}_{0\perp} t$. Such a current would generate a broadband radiation. For comparison, we write here the current density $\delta\mathbf{j}$ for the Raman regime

$$\delta\mathbf{j} = e n_b c^4 \mu \left(\frac{a_+}{A_0}\right)^2 \frac{k_0^2 \mathbf{k}_{0\perp} \sin(\alpha+\theta)}{\omega_b \omega_+ \gamma_0^{3/2}} t, \quad 0 < t < t_{\max}. \quad (45)$$

The estimation is $\delta\mathbf{j} \sim n_b^{1/2} \mu \gamma_0^{-3/2} \sin(\alpha+\theta) \mathbf{k}_{0\perp} t$. The estimations presented above indicate that of the two regimes, the Raman one is preferable for the ultrarelativistic electron beam, when $\omega_b/(k_0 c) \gg \sqrt{\gamma_0} \sin(\alpha+\theta)$.

The modulation of electron density considered above can lead to the formation of beam filaments with spatial period $\lambda_b$ in the plane perpendicular beam velocity during the nonlinear stage of amplification. It is also necessary to take the transverse modulation of the electron density into account for the linear stage of amplification in FELWI: its effect on electron's trajectories in the drift region between two wigglers is of the same order of magnitude as the field-induced angular variation of electron velocities passing the first wiggler.

## IV. NUMERICAL SIMULATION

To study development of the FEL instability in a noncollinear wiggler in more detail, we simulated the dynamics of the electron beam and the amplified radiation. For numerical simulation of amplification by an ultrarelativistic beam it is convenient to use the momentum evolution equation

$$\dot{\mathbf{p}} = -\frac{m}{2\gamma}\left(\frac{e}{mc}\right)^2 \nabla_\parallel A^2 - e\nabla_\parallel \phi \quad (46)$$

instead of Eq. (13). We introduce the dimensionless values

$$a = \frac{A_+}{A_0}, \quad \tau = k_0 c t, \quad \mathbf{Q} = \frac{\mathbf{p}}{mc}. \quad (47)$$

In addition we use orthogonal dimensionless coordinates $(\xi, \eta)$. The coordinate $\xi$ is defined above as the coordinate along $\mathbf{k}_0$. The coordinate $\eta$ directs along $\mathbf{k}_0 \times \mathbf{e}_y$, i.e., $\eta = (\mathbf{k}_0 \cdot \mathbf{e}_y \cdot \mathbf{r}_\parallel)$. As a result, the number of equations reduces, since the coordinate $\eta$ is ignorable. The equations of electron motion are

$$\frac{dQ_\xi}{d\tau} = -\frac{i}{2}\left[\mu \frac{a}{\gamma} + \left(\frac{\omega_b}{k_0 c}\right)^2 \sigma\right] e^{i\xi} + \text{c.c.}, \quad (48)$$

$$\frac{d\xi}{d\tau} = \frac{Q_\xi}{\gamma}, \quad (49)$$

$$Q_\eta = Q_\eta(0). \quad (50)$$

Here $\mathbf{Q} = \mathbf{Q}_\xi + \mathbf{Q}_\eta$,

$$\gamma = \sqrt{Q_\xi^2 + Q_\eta^2 + 1 + \mu(1 + a e^{i\xi} + \text{c.c.})}. \quad (51)$$

The dimensionless equation for the laser field takes the form

$$\frac{d^2 a}{d\tau^2} + \left(\frac{\omega_+}{k_0 c}\right)^2 a = -\frac{1}{2}\left(\frac{\omega_b}{k_0 c}\right)^2 \hat{\sigma}, \quad (52)$$

where

$$\left(\frac{\omega_+}{k_0 c}\right)^2 = 1 - 2\frac{k_w}{k_0}\cos\theta + \left(\frac{k_w}{k_0}\right)^2 + \left(\frac{\omega_b}{k_0 c}\right)^2 \frac{1}{2\pi}\int_0^{2\pi} \frac{d\xi_0}{\gamma} \quad (53)$$

and $\sigma$, $\hat{\sigma}$ are given by Eq. (11). The equations of motion are supplemented by the initial conditions $0 \le \xi_0 \le 2\pi$. In order to calculate the integrals in Eqs. (11) and (53) the values $\xi_0$ are put uniformly in the interval $[0, 2\pi]$

$$\xi_0^{(n)} = \frac{\pi}{N} n, \quad n = 0, 1, 2, \ldots, 2N. \quad (54)$$

Besides, the initial of momentum components are given by

$$Q_\xi(0) = \gamma_0 \frac{u}{c}\cos(\alpha+\theta), \quad Q_\eta(0) = \gamma_0 \frac{u}{c}\sin(\alpha+\theta). \quad (55)$$

The input parameters of calculation are dimensionless amplitude $\mu$ and period $k_w/k_0$ of undulator field, the dimensionless current $\mu_1 = \omega_b/(k_0 c)$, and velocity $\beta$ (or relativistic factor $\gamma_0$) of beam, and the angles $\alpha$ and $\theta$. For the resonant case of Eq. (24) the value $k_w/k_0$ is determined by $\beta$ and $\mu_1$

$$1 - 2\frac{k_w}{k_0}\cos\theta + \left(\frac{k_w}{k_0}\right)^2 = \left(\beta\cos(\alpha+\theta) - \mu_1\sqrt{\frac{1-\beta^2\cos^2(\alpha+\theta)}{\gamma_0}}\right)^2 - \frac{\mu_1^2}{\gamma_0}. \quad (56)$$

We introduce the dimensionless amplitude of beam potential wave

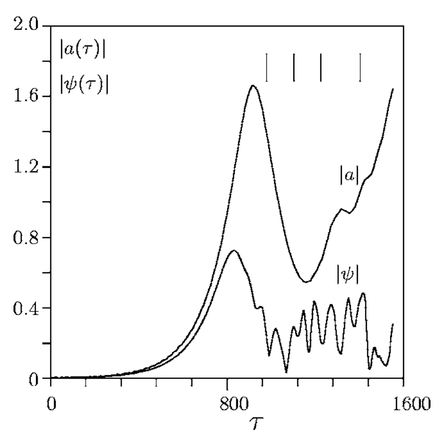


FIG. 3. The evolution of fields amplitudes for Raman regime, when $|q| = 0.051$. The vertical lines denote the moments of time, for which beam phase planes are presented in Fig. 4.

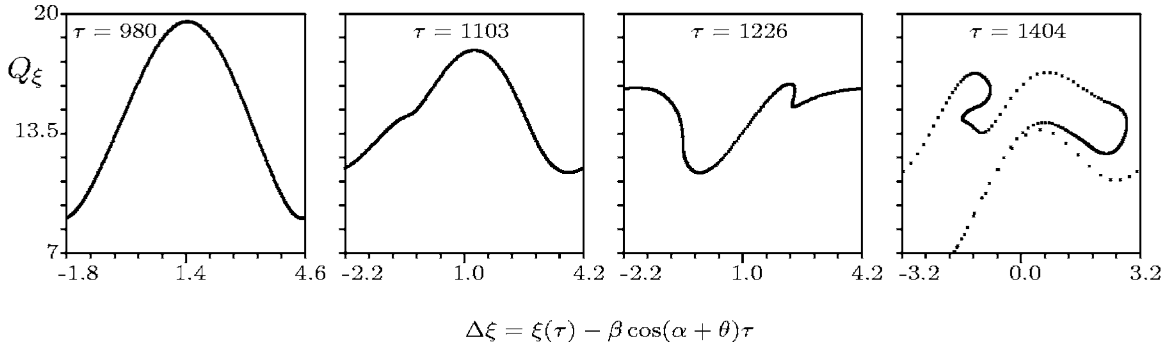


FIG. 4. The phase planes of the beam in different moment of time for Raman type: $|q|=0.051$.

$$\psi = \frac{m}{e}\frac{\omega_b^2}{k_0^2}\frac{\sigma}{A_0} = \sqrt{\frac{2}{\mu}}\mu_1^2\sigma. \tag{57}$$

Now let us discuss *saturation effects*, since namely for the saturated amplification the difference between single-electron and collective amplifications is prominent. In all calculations $\alpha=\theta=0.1$.

In Fig. 3 the evolution of field amplitudes for the Raman regime is when the FEL parameters a set to $\mu=0.5$, $\mu_1=0.9$, and $\gamma_0=15$. The resonant frequency is equal to $\omega_+=0.927k_0c$, while the dimensionless growth rate of instability is equal to $\mathrm{Im}(\delta\omega/k_0c)=0.008$. The parameters equal $|q|=0.051$ and $\Omega_b/k_0c=0.0498$. The saturation of instability in this case is shown by Fig. 4 to be a result of electrons deceleration and as a consequence violation of resonant condition $\omega=\mathbf{k}\cdot\mathbf{u}$ (nonlinear shift of frequency). The subsequent development of instability (for $\tau>1200$) leads to self-capture and reflection of electrons from crest of wave potential. At this stage numerous streams are formed leading to randomization of the beam (chaos of beam) for $\tau>1400$.

In Fig. 5 the evolution of the field amplitudes is present in the single-electron amplification case, when $\mu=2$, $\mu_1=0.01$, $\gamma_0=10$. For this case $\omega_+=0.964k_0c$, $|q|=6.42$, $\Omega_b/k_0c=0.000826$, and $\mathrm{Im}(\delta\omega/k_0c)=0.00133$. The saturation of amplification results from the capture of electrons by field amplified. Subsequent oscillations of the captured electrons lead to randomization of the beam (Fig. 6).

## V. CONCLUSIONS

In summary, we have studied the Thompson and Raman regimes of the FEL amplification for the noncollinear geometry of the electron and laser beams. For the first time, we have found the growth rates of FEL instability for both regimes as a function of electron relativity, beam density, and the angle between the electron and the laser beams. The conditions of Raman and Thompson excitation regimes have been obtained. We have found that for noncollinear arrangement the collective amplification is possible for any value of wiggler parameter $\mu$ or for any lateral relativistic velocity of electrons as distinct from the collinear wiggler arrangement, for which Raman regime is absent for $\mu>1$ ([14] and references cited therein). It has been found that the noncollinear geometry favors the collective (Raman) conditions for the amplification. It has also been found that if the wiggler is filled with the noncollinear electron beam and the laser wave, then Raman-type amplification is feasible for relatively small densities of the electron beam [the first growth rate in Eq. (33)], since the electron current density required for the Raman-type amplification drops with the increasing relativistic factor $\gamma_{\parallel 0}$ of the beam. This means that *collective amplification* can be realized exactly in optical wigglers, in particular, in a FELWI, which employs a relativistic electron beam noncollinear to a laser wave. We have found that for parameters used in the paper [8] to design numerically the FELWI, the Raman regime holds for beam current exceeding 10 A.

We have demonstrated that noncollinear interaction geometry leads to efficient generation of the second harmonic in the current density. We have found that of the two excitation regimes, the Raman one is preferable for generating CSR by ultrarelativistic electron beams, when $\omega_b/(k_0c)\gg\sqrt{\gamma_0}\sin(\alpha+\theta)$.

It has been found that the modulation of the beam density leads to formation of the transverse bunching with spatial period $\lambda_b$ [Eq. (41)]. We suppose that it is necessary to take this effect into account for the linear stage of amplification in a FELWI because its effect on electron's trajectories in the drift region between two wigglers is of the same order of magnitude as the field-induced angular variation of electron velocities passing the first wiggler for Raman regime.

Using computer simulations it was shown that for Raman amplification, the saturation results from electron decelera-

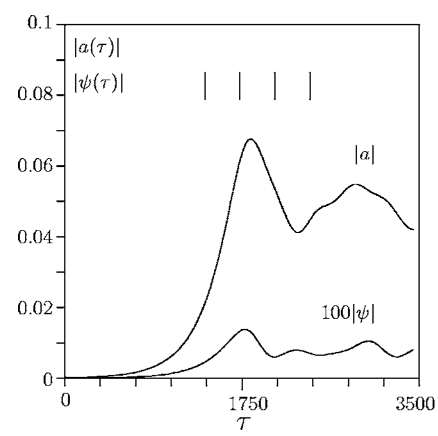


FIG. 5. The evolution of fields amplitudes for Thompson regime, when $|q|=6.42$. The vertical lines denote the moments of time, for which beam phase planes are presented in Fig. 6.

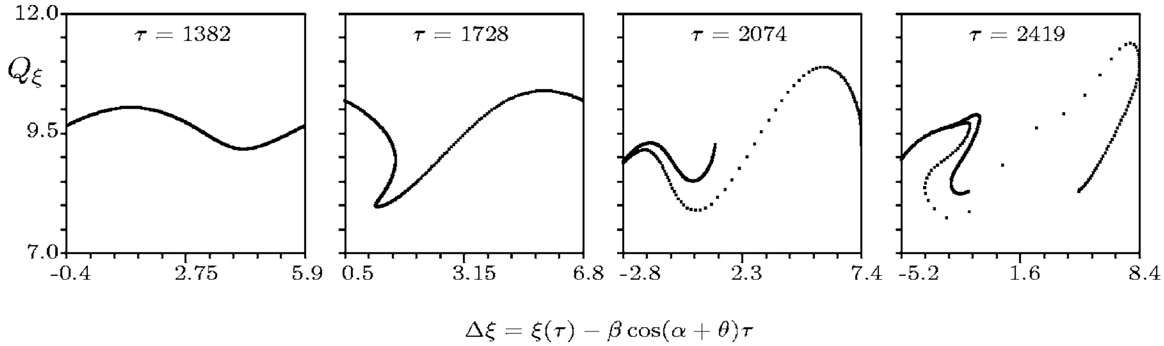


FIG. 6. The phase planes of the beam in a different moment of time for Thompson type: $|q|=6.42$.

tion, which leads to violation of the resonant condition $\omega = \mathbf{k}\cdot\mathbf{u}$ (nonlinear shift of the resonant frequency), meanwhile for Thompson type of amplification the saturation results from the capture of electrons by the field being amplified.

### ACKNOWLEDGMENTS

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### APPENDIX A: SOLUTION OF VLASOV EQUATION

The solution of the Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = 0 \tag{A1}$$

can be represented in the form

$$f(t,\mathbf{r},\mathbf{p}) = \int\int f_0(\mathbf{p}_0)\delta[\mathbf{r} - \mathbf{r}(t,\mathbf{r}_0,\mathbf{p}_0)] \tag{A2}$$

$$\delta[\mathbf{p} - \mathbf{p}(t,\mathbf{r}_0,\mathbf{p}_0)]d^3\mathbf{r}_0 d^3\mathbf{p}_0.$$

Here $\mathbf{F}$ is a force acting on the electrons, $f_0(\mathbf{p}_0)$ is an initial distribution function for beam electrons; $\mathbf{r}(t,\mathbf{r}_0,\mathbf{p}_0)$ and $\mathbf{p}(t,\mathbf{r}_0,\mathbf{p}_0)$ are solutions of the characteristic system (4) for the Vlasov equation, being Hamilton equations; $\mathbf{r}_0$ and $\mathbf{p}_0$ are the initial coordinates.

For the monoenergy electron beam the initial distribution function is $f_0=n_b\delta(\mathbf{p}_0-m\gamma_0\mathbf{u})$. In this case it is easy to obtain the perturbed parts for charge and current densities, namely Eqs. (3).

### APPENDIX B: SOLUTION OF EQ. (31)

The solutions of Eq. (31) are $\delta=\frac{1}{4}|p|\pm\frac{1}{2}\sqrt{\mathcal{D}}$, where

$$|p| = \frac{1}{2}\frac{\mu}{1+\mu}\frac{\omega_b^2}{\Omega_b^2\gamma_0\gamma_{\|0}^2} = \frac{\nu}{(1+\nu)^2}\left(\frac{\mathbf{k}_0\cdot\mathbf{u}}{k_0 c}\right)^2|q| \tag{B1}$$

and

$$\mathcal{D} = -2|q|\left(1 - \frac{1}{8}\frac{\mu}{1+\mu}\frac{\nu}{(1+\nu)^2}\left(\frac{\mathbf{k}_0\cdot\mathbf{u}}{k_0 c}\right)^2\frac{\omega_b^2}{\Omega_b^2\gamma_0\gamma_{\|0}^2}\right). \tag{B2}$$

Since the second term in brackets is smaller than 1/32, it can be neglected. The solution of Eq. (31) takes the form

$$\delta = \frac{1}{4}|p| \pm i\sqrt{\frac{|q|}{2}}, \tag{B3}$$

from which we obtain the growth rate (32).

### APPENDIX C: SOLUTION OF EQ. (35)

The solution of Eq. (35) for the imaginary part of $\delta$ is

$$\mathrm{Im}(\delta) = \frac{\sqrt{3}}{2}\left(\frac{|q|}{2}\right)^{1/3} f(\kappa). \tag{C1}$$

Here function $f(\kappa)$ is

$$f(\kappa) = \left(\sqrt{1+\kappa^{-2}}+1\right)^{1/3} + \left(\sqrt{1+\kappa^{-2}}-1\right)^{1/3} \tag{C2}$$

and its argument is

$$\kappa = \frac{3\sqrt{3}}{2\sqrt{2}}\frac{(1+\nu)^2}{\nu}\left(\frac{k_0 c}{\mathbf{k}_0\cdot\mathbf{u}}\right)^2 \times \sqrt{\left(1+\frac{1}{\mu}\right)\left[1+(\gamma_{\|0}^2-1)\sin^2(\alpha+\theta)\right]}. \tag{C3}$$

Since $\kappa \geqslant 3\sqrt{6}\left(\frac{k_0 c}{\mathbf{k}_0\cdot\mathbf{u}}\right)^2 > 3\sqrt{6}$, we obtain $f(\kappa)=2^{1/3}$ for asymptote $\kappa^{-2} \ll 1$, and the solution (36).

[1] J. M. J. Madey, Nuovo Cimento Soc. Ital. Fis., B **50B**, 64 (1979).
[2] C. A. Brau, *Free-Electron Lasers* (Academic, Boston, 1990).
[3] G. Kurizki, M. O. Scully, and C. Keitel, Phys. Rev. Lett. **70**, 1433 (1993).
[4] B. Sherman, G. Kurizki, D. E. Nikonov, and M. O. Scully, Phys. Rev. Lett. **75**, 4602 (1995).
[5] D. E. Nikonov, B. Scherman, G. Kurizki, and M. O. Scully, Opt. Commun. **123**, 363 (1996).
[6] D. E. Nikonov, M. O. Scully, and G. Kurizki, Phys. Rev. E **54**, 6780 (1996).
[7] D. E. Nikonov, Yu. V. Rostovtsev, and G. Sussmann, Phys. Rev. E **57**, 3444 (1998).
[8] Yu. Rostovtsev, S. Trendafilov, A. Artemiev, K. Kapale, G. Kurizki, and M. O. Scully, Phys. Rev. Lett. **90**, 214802 (2003).
[9] A. A. Varfolomeev and T. V. Yarovoi, Nucl. Instrum. Methods Phys. Res. A **445**, 290 (2000); **429**, 435 (1999).
[10] W. B. Colson, G. Dattoli, and F. Ciocci, Phys. Rev. A **31**, 828 (1985).
[11] M. Xie, Nucl. Instrum. Methods Phys. Res. A **483**, 527 (2000).
[12] H. K. Avetissian, H. A. Jivanian, and R. G. Petrossian, Phys. Lett. **66**, 161 (1978).
[13] A. A. Varfolomeev, S. V. Tolmachev, A. Varfolomeev Jr., and T. V. Yarovoi, *Proceedings of RuPAC XIX* (Dubna, Nauka, 2005), p. 79; A. A. Varfolomeev and Yu. Yu. Lachin, Nucl. Instrum. Methods Phys. Res. A **296**, 411 (1990).
[14] M. V. Kuzelev and A.A. Rukhadze, *Plasma Free Electron Lasers* (Edition Frontier, Paris, 1995).
[15] A. B. Matsko and Yu. V. Rostovtsev, Phys. Rev. E **58**, 7846 (1998).